Flow Under a Sheet Pile

Revised: 11/04/07 jcw

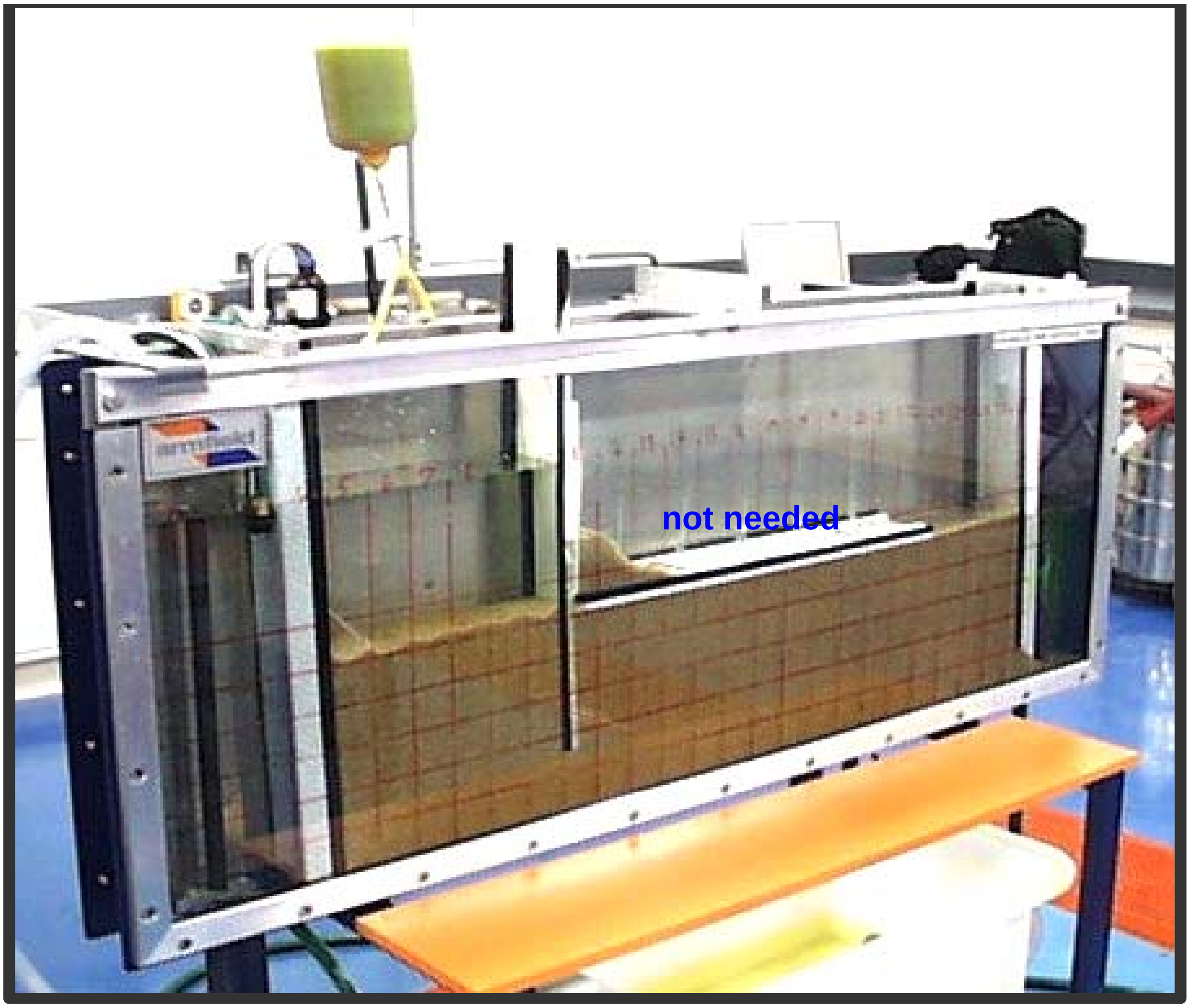


Figure 1. Picture of experimental setup. Note: you don't need to include the horizontal section in your experiment and the sand should be higher.

Purpose:

This laboratory exercise in intended to investigate steady unconfined groundwater flow under a piling. Solution of equations using the finite difference method is introduced.

Background:

In a homogeneous, isotropic system at steady state, groundwater flow can be modeled with Laplace's Equation:

Draft Dated: November 4, 2007

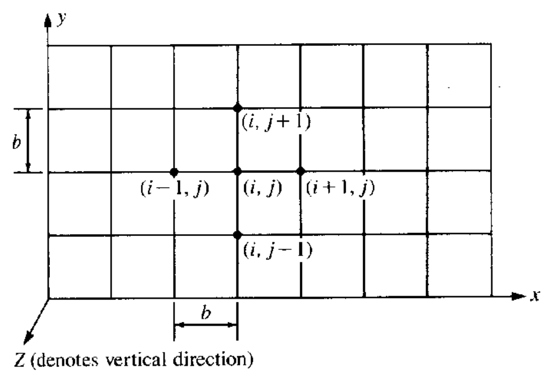
$$\nabla^2 h = 0 \tag{1}$$

Using the method of finite difference, Laplaces Equation can be solved iteratively using:

$$h_{i,j} = \frac{(h_{i+1,j} + h_{i-1,j} + h_{i,j+1} + h_{i,j-1})}{4} \tag{2}$$

where the value of the hydraulic head at any point is the sum of the head at the four adjacent computational nodes.

The i and j refer to node locations on a two dimensional grid of nodes. At the edge or boundary of the system a different equation must be used. From a mathematics perspective, and in the nomenclature of partial differential equations, the edges of the system are described with boundary conditions. In these experiments we have two types of boundary conditions a) fixed hydraulic head and b) no flow. Fixed head boundary conditions are found on the left hand side and at the river. No flow boundary conditions are found on the top, bottom, and right hand sides. If datum is taken to be the bottom, then the hydraulic head on the sides is equal to the height of the water at the top of the reservoirs. The river also represents a fixed head boundary condition.



Equation 1 uses a numerical technique called relaxation. The equation is applied iteratively at each node, until eventually, the answers “relax” to the correct solution. The analogy is somewhat like my waistline. Each year it seems to relax a little further under the effects of gravity.

Analysis

Using an Excel spreadsheet, make a numerical model of the experiments and compare the numerical model with experimental results. Make nodal spacings at a 2 cm distance. The slope of the sand at the top of the apparatus can be ignored in the simulation model. The interior nodes are calculated using Eq (1). Fixed head boundary conditions are simply specified as a constant value in the spreadsheet. No flow boundaries are specified by reflection. If we have a no flow boundary, then the flow attempting to cross the boundary is merely reflected back. Mathematically, this can

Draft Dated: November 4, 2007

be obtained by ignoring the node just outside the grid and double counting the node along the same direction inside the grid. For example, nodes along the bottom can be calculated as:

$$h_{i,j} = \frac{(h_{i+1,j} + h_{i-1,j} + 2h_{i,j+1})}{4} \tag{3}$$

After putting equations in the spreadsheet for all interior and boundary nodes, instruct the spreadsheet to iterate till convergence. This can be specified under **Options**, **Calculation**.

After the spreadsheet has converged, print out the values at each node. The results represent the hydraulic head at each node. Unfortunately we require the flow direction. I guess we'll have to do some more work! We know that the flow rate is given as:

$$V = -K\nabla h \tag{4}$$

so we must find the slope or gradient of hydraulic head. Any parcel of water always travels down the gradient of hydraulic head, just a marble rolls down hill. Draw a contour map of equal values of hydraulic head. Begin at the fixed head boundary on the left side and draw several flow paths. The flow line should always cross the hydraulic head contours at a 90 degree angle (i.e, the marble always rolls down the steepest direction).

Methods:

i Break into groups of 3 students and perform separate experiments for each group.

ii Set up the experiment as shown in the figure, except that the horizontal piece is not needed, just the sheet pile. The sand should also be filled to nearer to the top of the apparatus. Mix up a batch of dye.

iii Common problems with this lab are leakage of water through the sheet pile and inability to get good dye traces. Make sure to get a good seal between the sheet pile and the glass by eliminating sand at the contact point.

iv Make a grid on the glass using ***erasable*** marker.

v Add water to the upper end and set the drain at the appropriate level in the Drainage and Seepage Tank. Measure and record the water level at each end of the tank and the water table across the system.

vi Let the system come to steady state (occurs in a couple of minutes).

vii By moving the injection point, create at least 4 dye traces. Record the path of each dye trace using the marked grid. Place the tubes with the dye near or against the glass

Draft Dated: November 4, 2007

to facilitate visual control of the dye injection rate. The air should initially be purged from the dye injection tubes. Once the tubes are filled with water dye can be put in a few drops at a time by dripping into the top of the tubes.

Analysis

i Draw a schematic of the system to scale and include this figure in your laboratory report?

ii Develop a finite difference model for the system using Excel. Make each cell square shaped in Excel.

iii Use your model to predict the distribution of hydraulic head in the system.

iv Print out the hydraulic heads.

v By hand, draw contours of constant hydraulic head.

vi By hand, draw several flow lines.

vii Draw the measured dye traces on your Excel printout.

viii Comment on the relationship between measured and modeled values.

Applied Hydrogeology - Fourth Edition, C.W. Fetter, Prentice-Hall, 2001.

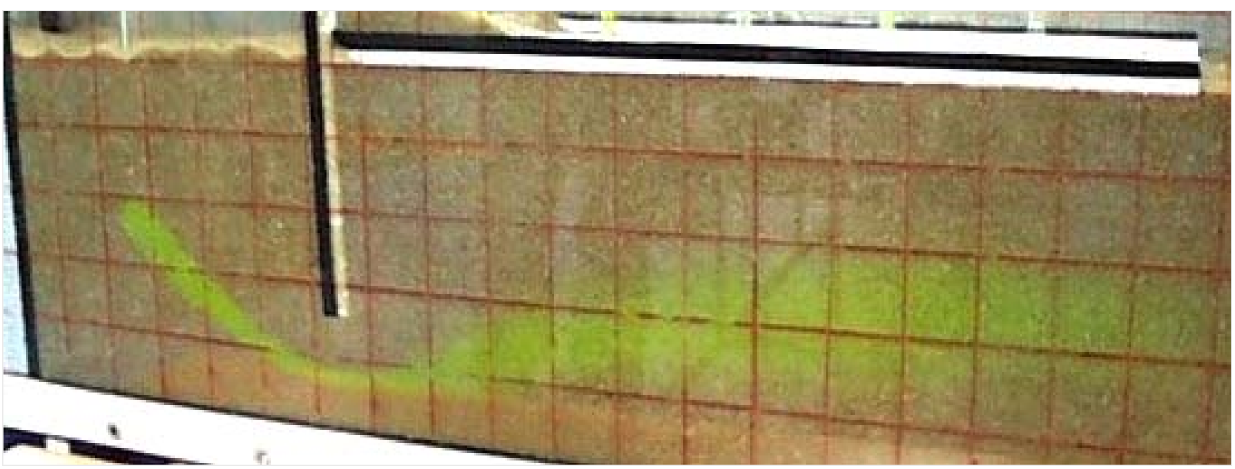


Figure 2. Example of a dye trace. Notice how the stream tube contracts and then expands. Why does it do that?

Draft Dated: November 4, 2007